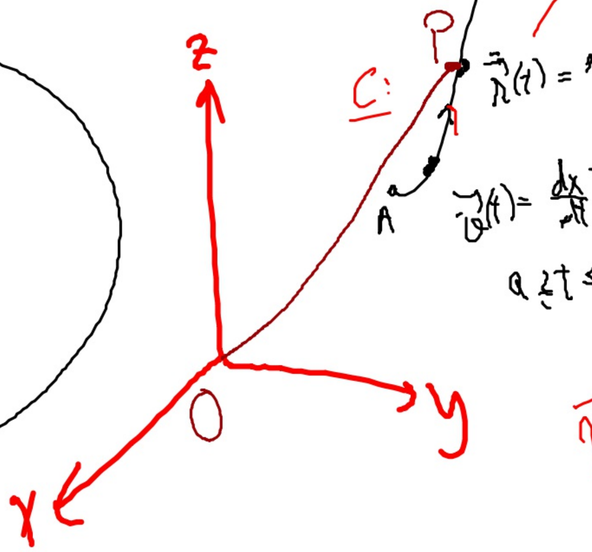
§ 16.1 Line integral

distance along a curve $\longleftarrow$ $s(t) = \int_a^t |\vec{v}|\, d\tau$ $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}$

$ds = |\vec{v}(t)|\, dt$



"= position vector"

$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$

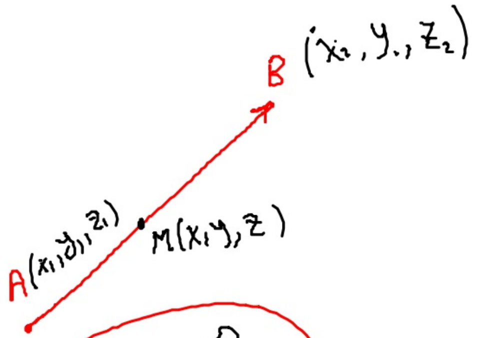
$\vec{v}(t) = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$

$a \le t \le b$

$$\int_C f(x,y,z)\, ds = \int_a^b f(t)\, |\vec{v}|\, dt$$

$$\vec{v} = \frac{d\vec{r}}{dt}$$

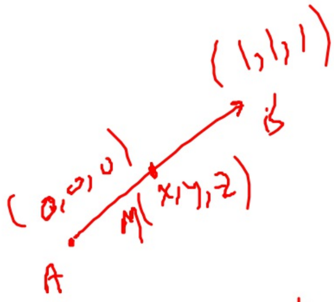
Line segment



$$\overrightarrow{AM} = t\,\overrightarrow{AB}$$
$$0 \le t \le 1$$

$$\overrightarrow{AM} = \begin{vmatrix} x - x_1 \\ y - y_1 \\ z - z_1 \end{vmatrix}$$

$$\overrightarrow{AB} = \begin{vmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{vmatrix}$$

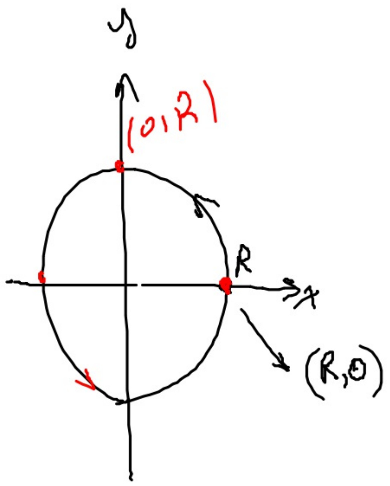


$$\overrightarrow{AM} = t\,\overrightarrow{AB} \;;\; 0 \le t \le 1$$

$$\overrightarrow{AM} = \begin{vmatrix} x \\ y \\ z \end{vmatrix} \qquad \overrightarrow{AB} = \begin{vmatrix} 1 \\ 1 \\ 1 \end{vmatrix}$$

$$x = y = z = t$$
$$0 \le t \le 1$$

Circles : $x^2+y^2=R^2$



Counterclockwise:

$x = R\cos t$

$y = R\sin t$

$0 \le t \le 2\pi$

clockwise:

$x = R\cos t$

$y = -R\sin t$

$0 \le t \le 2\pi$

$(x-h)^2+(y-k)^2 = R^2$

Counterclockwise

$x-h = R\cos t$

$y-k = R\sin t$

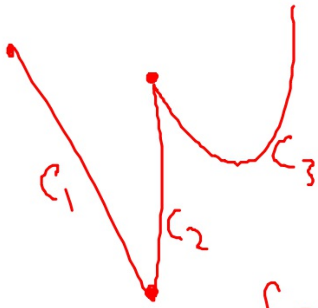
$0 \le t \le 2\pi$

$y = f(x)$

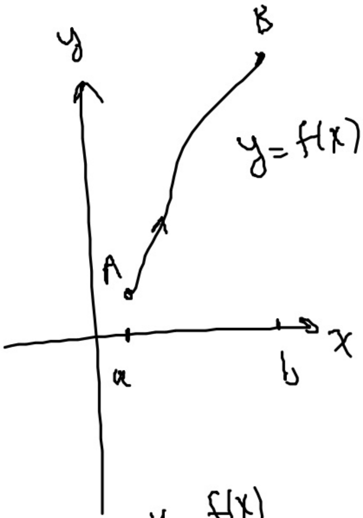
or

$x = g(y)$

C :



$$\int_C f\,ds = \int_{C_1} f\,ds + \int_{C_2} f\,ds + \int_{C_3} f\,ds$$



$y = f(x)$

$x = t$ & $y = f(t)$

$a \leq t \leq b$

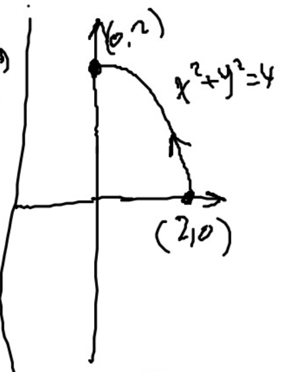
29) $\int_C (x+y)\, ds$

$= \int_0^{\frac{\pi}{2}} (2\sin t + 2\cos t)\, 2\, dt$

$= \dots$

$ds = |\vec{v}|\, dt$

$\vec{r}(t) = x(t)\,\vec{i} + y(t)\,\vec{j}$



$x = 2\cos t$

$y = 2\sin t$

$0 \le t \le \frac{\pi}{2}$

$\vec{v} = -2\sin t\,\vec{i} + 2\cos t\,\vec{j}$

$|\vec{v}| = 2$

10) $\int_C (x-y+z-2)\,ds$

$= \int_0^1 (t+t-1+1-2)\sqrt{2}\,dt$

$= \sqrt{2}\int_0^1 (2t-2)\,dt$

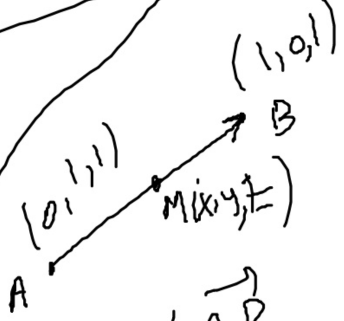
$= - - - -$

$x=t \quad y=1-t \quad z=1$

$0 \le t \le 1$

$\vec{v} = \vec{i} - \vec{j}$

$|\vec{v}| = \sqrt{2}$



$\vec{v} = \frac{d\vec{r}}{dt}$

$= \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$

$\vec{AM} = t\vec{AB}$

$0 \le t \le 1$

$\vec{AM} = \begin{vmatrix} x \\ y-1 \\ z-1 \end{vmatrix}$

$\vec{AB} = \begin{vmatrix} 1 \\ -1 \\ 0 \end{vmatrix}$

12) $\int_C \sqrt{x^2+y^2}\, ds$

$= \int_{-2\pi}^{2\pi} (4)(5)\, dt$

$= 20 \int_{-2\pi}^{2\pi} dt = 80\pi.$

$\vec{r}(t) = 4\cos t\, \vec{i} + 4\sin t\, \vec{j} + 3t\, \vec{k}$

$-2\pi \le t \le 2\pi$

(Helix)

$ds = |\vec{v}|\, dt$

$\vec{v} = -4\sin t\, \vec{i} + 4\cos t\, \vec{j} + 3\vec{k}$

$|\vec{v}| = \sqrt{25} = 5$

27 $\int_C \frac{x^3}{y}\, ds$

$C: y = \frac{x^2}{2}$

$$= \int_0^2 \frac{t^3}{\frac{t^2}{2}} \sqrt{1+t^2}\, dt$$

$$= \int_0^2 2t\sqrt{1+t^2}\, dt$$

$$\boxed{\begin{array}{l} \text{let } u = 1+t^2 \\ du = 2t\, dt \end{array}}$$

$$= \int_1^5 \sqrt{u}\, du = \left. \frac{2}{3} u^{\frac{3}{2}} \right|_1^5 = \cdots$$

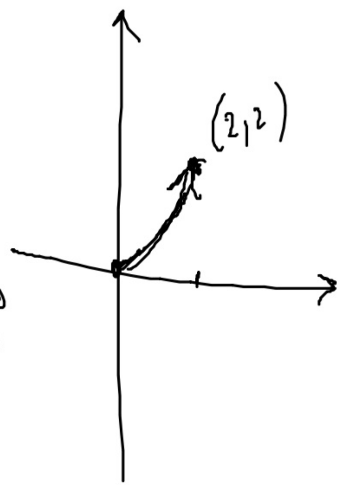
$x = t$

$y = \frac{t^2}{2}$

$0 \le t \le 2$

$\vec{v} = \vec{i} + t\vec{j}$

$|\vec{v}| = \sqrt{1+t^2}$



(15) $\int_C (x+\sqrt{y}-z^2)\,ds = \int_{C_1} \cdots + \int_{C_2}$

$$= \int_0^1 (t+t)\sqrt{1+4t^2}\,dt$$

$$+ \int_0^1 (1+1-t^2)\,dt$$

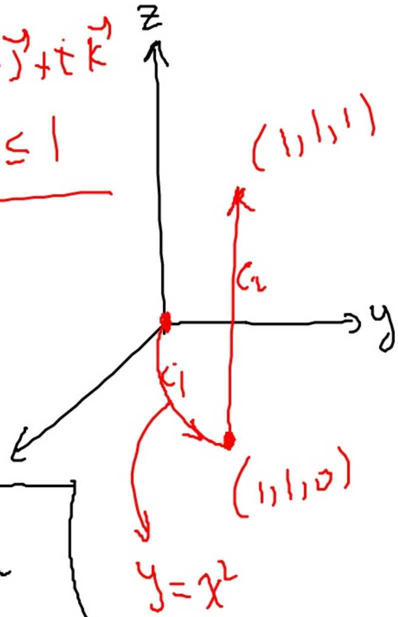
$$= \cdots$$

$\vec{r}_2(t) = \vec{i} + \vec{j} + t\vec{k}$

$0 \le t \le 1$

$\vec{v}_2 = \vec{k}$

$|\vec{v}_2| = 1$



$x = t$
$y = t^2$

$\vec{r}_1 = t\vec{i} + t^2\vec{j}$

$0 \le t \le 1$

$\vec{v}_1 = \vec{i} + 2t\vec{j}$.